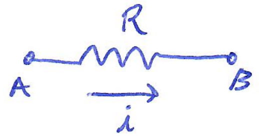
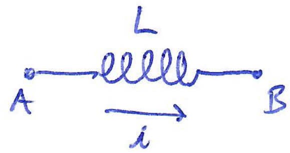
ME 4555 - Lecture 6 - Op-Amps

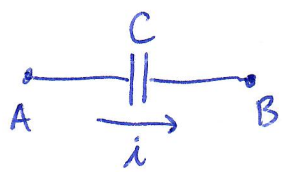
Reminder:



$V_A - V_B = iR$



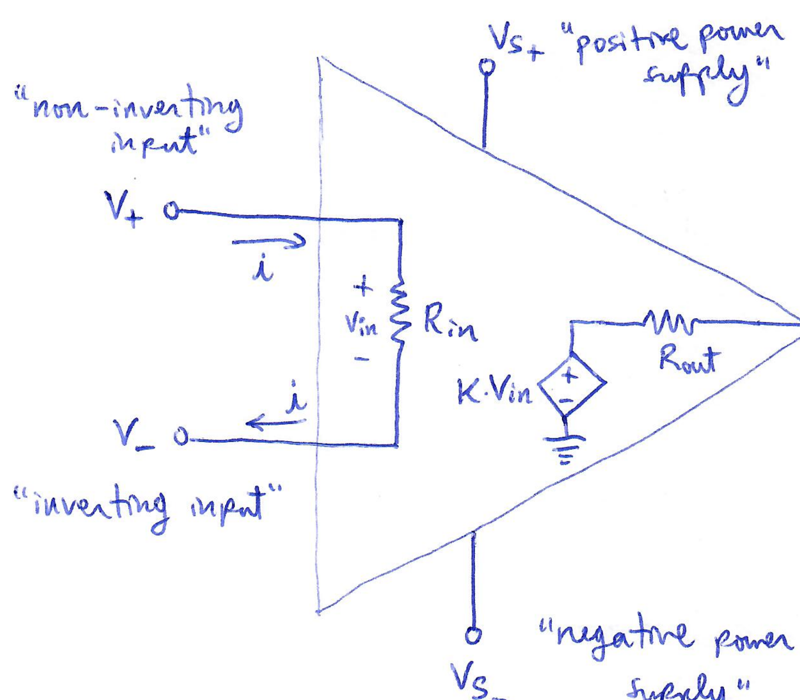
$V_A - V_B = L\dfrac{di}{dt}$



$V_A - V_B = \dfrac{1}{C}\int i\,dt$

passive elements. ideal inductor and capacitor are conservative whereas resistors are dissipative.

Today, we'll introduce operational amplifiers (opamps). Opamps are active elements (they can add energy to the system). Somewhat realistic model of an opamp:



Because of Rin:

$V_{in} = V_+ - V_-$

$V_{in} = i\,\underbrace{R_{in}}$

typically very large. so $i \approx 0$.

Because of voltage source,

$V_{out} = \underbrace{K} \cdot V_{in}$

typically very large, $K \approx 10^6$

Rout is typically small.

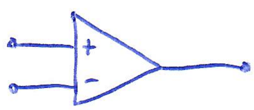
In a real opamp, the output is bounded by the supply. So

$V_{S_-} \leq V_{out} \leq V_{S_+}$

In an <u>ideal</u> opamp, $R_{in} = \infty$, $K = \infty$, $R_{out} = 0$, $V_{S+} = +\infty$, $V_{S-} = -\infty$. Therefore, $\boxed{i = 0}$ and $\boxed{V_+ = V_-}$

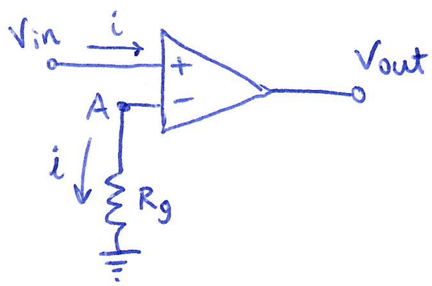
<u>Note</u>: V_{out} is not generally zero.

We usually represent an ideal opamp with the symbol:



<u>Note</u>: the "+" and "−" terminals are important! Sometimes we will flip the diagram to avoid crossed wires, so be mindful of the labels!

<u>Simple opamp circuit without feedback (open-loop)</u>: ("comparator").



since $\begin{cases} V_{out} = K \cdot (V_{in} - V_A) \\ V_A = i R_g, \quad V_{in} - V_A = i R_{in} \end{cases}$

K and R_{in} are the internal gain and input resistance of the opamp.

i) solve for V_A and eliminate i from last two equations:

$$V_A = i R_g = \frac{R_g}{R_{in}}(V_{in} - V_A) \Rightarrow V_A = \frac{R_g}{R_{in} + R_g} V_{in}.$$

ii) substitute into first equation:

$$V_{out} = K(V_{in} - V_A) = \frac{K}{1 + \frac{R_g}{R_{in}}} V_{in}$$

goes to $+\infty$ as $K \to \infty$ and $R_{in} \to \infty$.

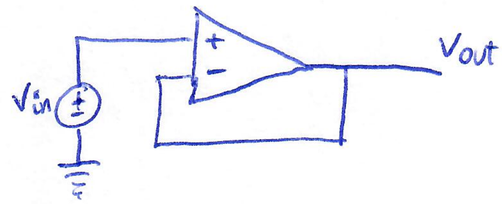
for an "ideal" opamp, the output would be $\pm\infty$. Here, we observe <u>saturation</u>:

In reality (since we can't have infinite voltages), the "ideal opamp" assumption <u>can't be used</u>. We observe instead:

$$\boxed{V_{out} = \begin{cases} V_{S+} & \text{if } V_{in} > 0 \\ V_{S-} & \text{if } V_{in} < 0 \end{cases}}$$

Negative vs Positive feedback

negative feedback uses the "−" input for feedback:



we have $V_{out} = K(V_{in} - V_{out}) \Rightarrow V_{out} = \frac{K}{1+K} V_{in}$

as $K \to \infty$, $\boxed{V_{out} = V_{in}}$

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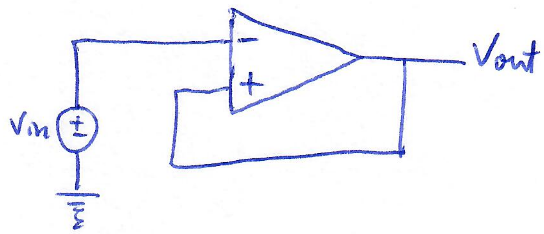
positive feedback uses the "+" input for feedback:



this is incorrect!!

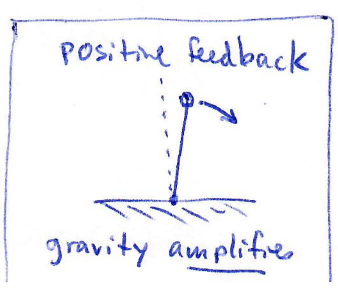
We have $V_{out} = K(V_{out} - V_{in}) \Rightarrow V_{out} = \frac{K}{K-1} V_{in}$

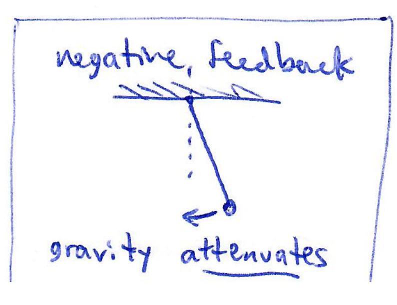
as $K \to \infty$, $\boxed{V_{out} = V_{in}}$

What's the problem? in negative feedback, the opamp works to keep $V_{in} = V_{out}$. If V_{in} increases slightly so $V_+ > V_-$, V_{out} (i.e. V_-) will increase to restore the balance. Likewise if $V_+ < V_-$, V_{out} will decrease to restore the balance.

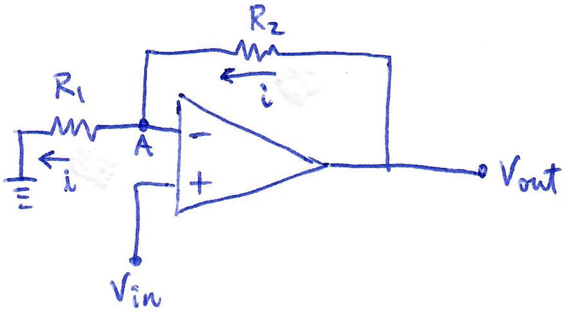
In positive feedback, small errors are amplified instead of attenuated. So such circuits are unstable and will cause saturation and the opamp will cease to behave ideally.

Analogy:





Ex: non-inverting amplifier



ideal opamp: ★ voltage input equal: $V_{in} = V_A$.

★ zero input current so both i's are same.

Voltage drop across resistors:

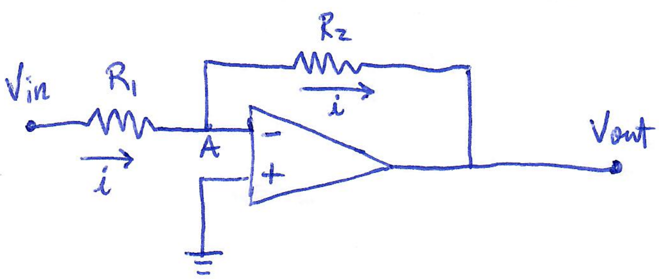
$$V_A = iR_1 \quad , \quad V_{out} - V_A = i R_2.$$

i) Eliminating i, we get: $V_{out} - V_A = iR_2 = V_A \cdot \frac{R_2}{R_1}$

ii) Substitute $V_A = V_{in}$: $V_{out} - V_{in} = \frac{R_2}{R_1} V_{in}$

iii) Solve for V_{out}: $\boxed{V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}}$ ← can tune R_1, R_2 to change how much output is amplified!

Ex: inverting amplifier



ideal opamp: ★ $V_A = 0$

★ both i's are the same.

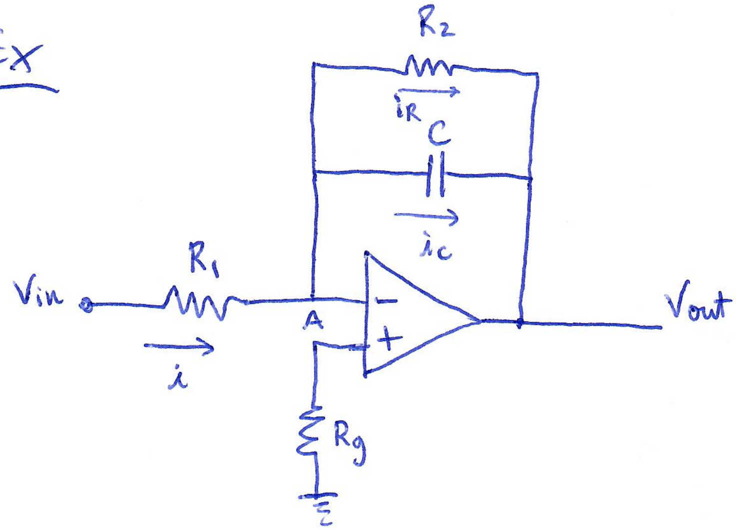
Voltage drop across resistors:

$$V_{in} - V_A = iR_1 \quad , \quad V_A - V_{out} = iR_2.$$

setting $V_A = 0$ and solving for V_{out} (eliminate i), we obtain:

$$V_{in} = iR_1 \quad , \quad V_{out} = -iR_2 \quad \Rightarrow \quad \boxed{V_{out} = -\frac{R_2}{R_1} V_{in}}$$

Ex



What is V_{out} in terms of V_{in}? (ideal opamp)

★ current into opamp input is zero, so by KCL: $i = i_C + i_R$.

★ input voltages is the same so $V_A = 0$.

For R_1: $V_{in} = i R_1$

For R_2: $-V_{out} = i_R R_2$

For C: $i_C = -C \dot{V}_{out}$

$$\xrightarrow{KCL} \underbrace{\frac{V_{in}}{R_1}}_{i} = \underbrace{-C\dot{V}_{out}}_{i_C} \underbrace{- \frac{V_{out}}{R_2}}_{+\, i_R}$$

$$\Rightarrow \boxed{C\dot{V}_{out} + \frac{1}{R_2} V_{out} = -\frac{1}{R_1} V_{in}}$$

if we make $R_2 \to \infty$ (i.e. make an open circuit; remove R_2 entirely)

we obtain $C\dot{V}_{out} = -\frac{1}{R_1} V_{in}$

$$\Rightarrow \boxed{V_{out} = -\frac{1}{R_1 C}\int V_{in}(t)\, dt.}$$

this is one way to construct an integrator!